

AMPLITUDE SHIFT

KEYING (ASK)

Digital Modulation Scheme.

$$p(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$
$$0 \leq t \leq T$$
$$T = \frac{k}{f_c}$$

$$T = \text{symbol Duration} \\ = \frac{k}{f_c}$$

integer number of
cycles in symbol
Duration

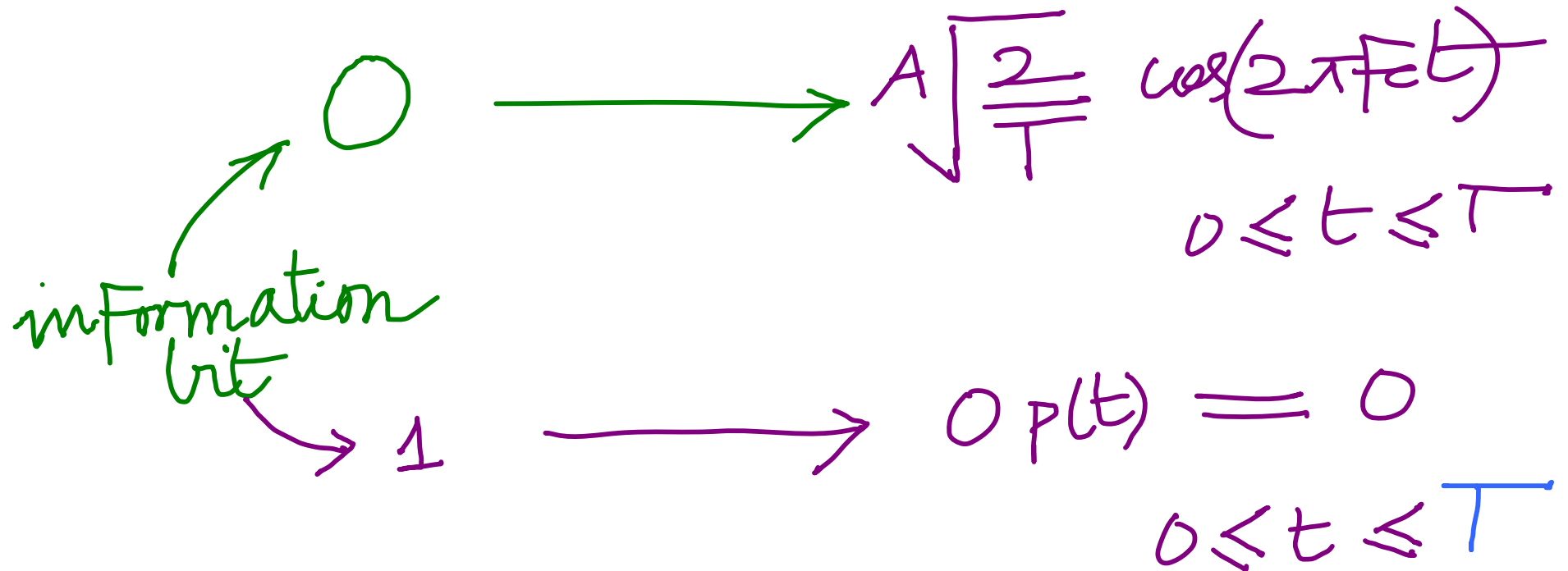
$$E_p = \int_0^T p^2(t) dt = 1$$

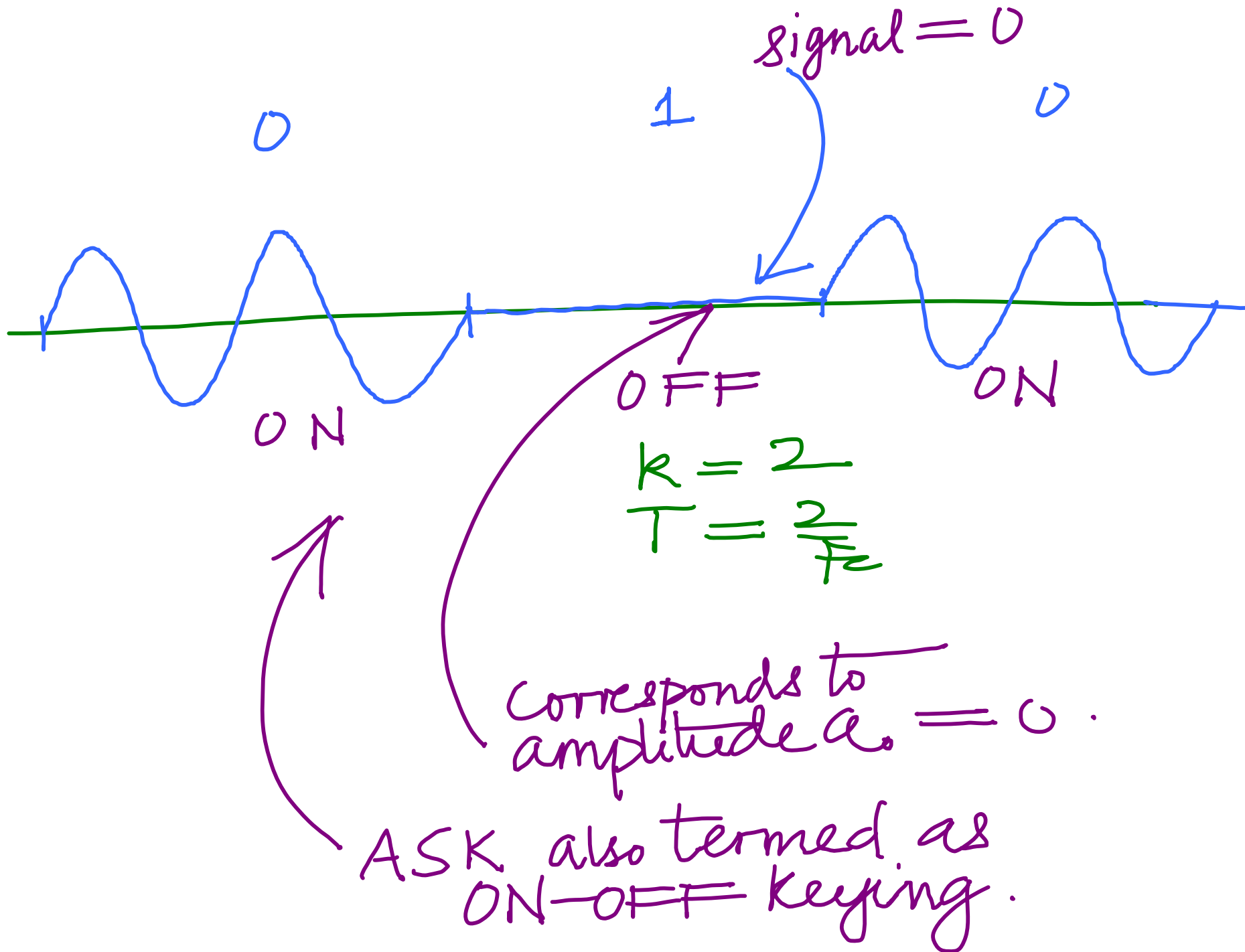
Energy of Pulse.

$$a_0 \in \{0, A\}$$

$$a_0 = \underline{A} \text{ or } \underline{0}$$

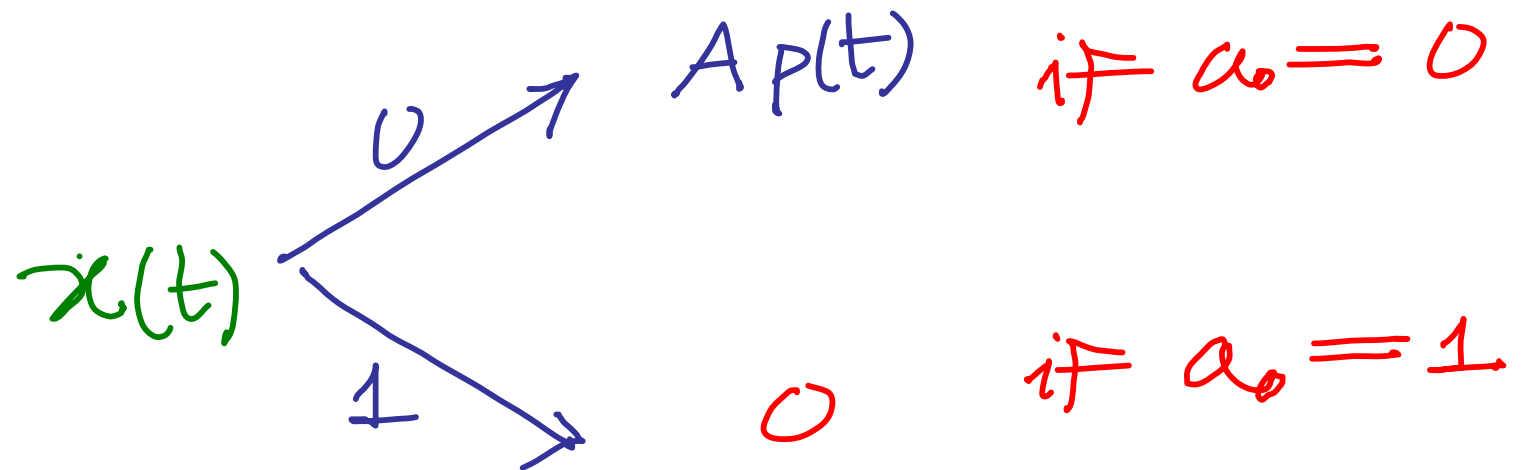
↑ waveforms differ in amplitude.





How to choose A?

Bit Energy = E_b .



$$Pr(0) = Pr(1) = \frac{1}{2}$$

Average energy per bit

$$= \frac{1}{2} \cdot A^2 E_p + 0 = \frac{1}{2} A^2 E_p.$$

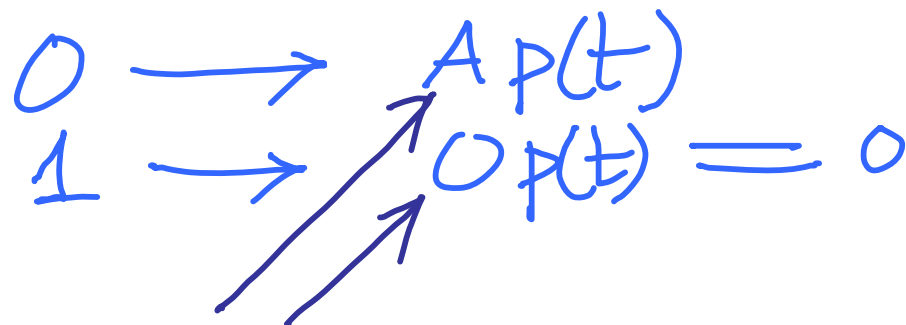
$$\frac{1}{2} A^2 \epsilon_p = \epsilon_b.$$


 $\epsilon_p = 1$

$$\Rightarrow \frac{1}{2} A^2 = \epsilon_b.$$

$$\Rightarrow \boxed{A = \sqrt{2\epsilon_b}}.$$

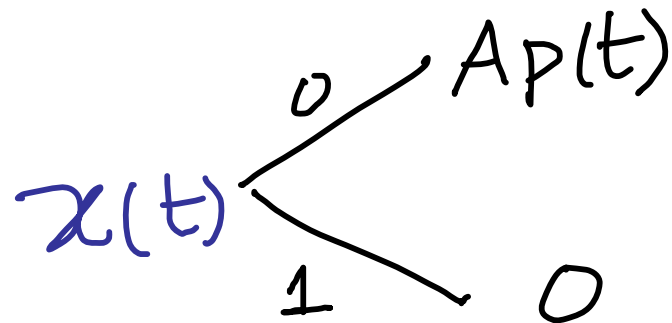
Ensures energy/bit
is constant across
schemes.



Both waveforms
differ in amplitude.

Therefore, this digital
modulation scheme is
termed as amplitude
shift keying. (ASK)

Prob of Error :



At the receiver

$$y(t) = x(t) + n(t)$$

AWGN
Additive White
Gaussian Noise

Matched Filter
with $h(t) = p(T-t)$

Followed by sampling
at $t = T$.

Consider bit = 0

$$y(t) = A p(t) + n(t)$$

After Matched Filtering,
followed by sampling
at $t = T$

$$r(T) = A E_p + \tilde{n}$$

Gaussian
mean = 0
var = $\frac{N_0 E_p}{2}$
= $\frac{N_0}{2}$

Consider information bit = 1

$$x(t) = 0, p(t) = 0$$

Not transmitting
any signal.

$$y(t) = 0 + n(t) \\ = n(t)$$

After MF & sampling
at $t = T$

$$r(T) = \tilde{n} \quad \text{--- Gaussian}$$

mean = 0
var = $\frac{N_0}{2} B_p$

$$r(t) = \begin{cases} \underline{A\epsilon_p + \tilde{n}} & \text{if } a_0 = A \\ \underline{\tilde{n}} & \text{if } a_0 = 0. \end{cases}$$

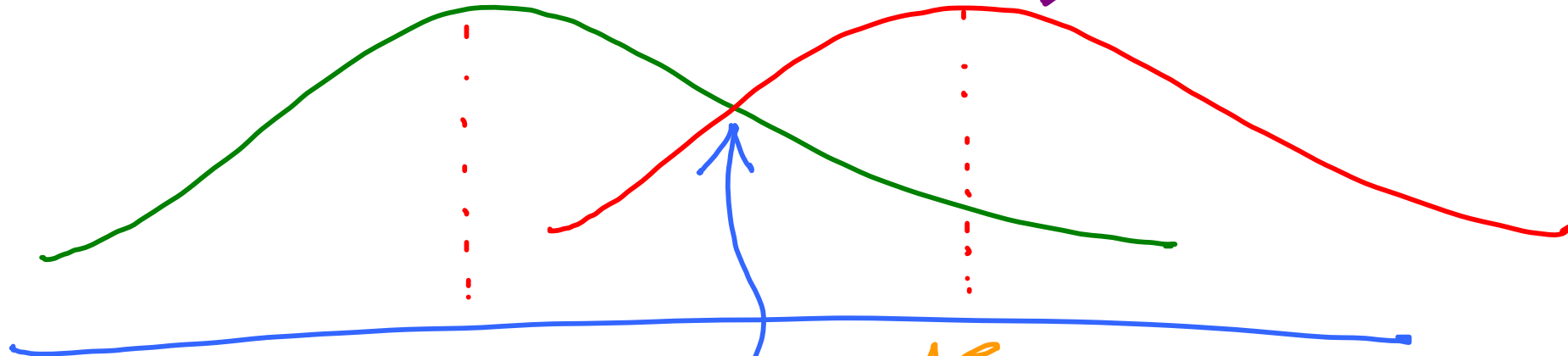
What is the optimal Decision rule?

Gaussian with mean = $A\epsilon_p$.

Gaussian with mean = 0.

Gaussian
for bit = 1

Gaussian
for bit = 0.



0
↑
bit = 1

$A\epsilon_p$
↑
bit = 0.

Detection
Threshold.

Symmetry

$$\text{midpt} = \frac{A\epsilon_p}{2}$$

Optimal Decision rule

$$\text{If } r(t) \geq \frac{A\epsilon_p}{2} \Rightarrow a_0 = A$$

$$\text{If } r(t) < \frac{A\epsilon_p}{2} \Rightarrow a_0 = 0$$

Optimal Decision rule — By symmetry